An Experimental Study of the Validity of Fourier's Law

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Speculation as to the validity of Fourier's law in describing heat conduction in materials subjected to highly nonuniform temperature fields has been advanced by several investigators (*1* to *3*). This speculation is prompted by the analogy with momentum transfer, where there is considerable evidence of nonlinearities associated with the flow and deformation of viscoelastic fluids. Very little is known about the nature and importance of nonlinear energy transfer (non-Fourier conduction). It has been only in the past few years that definite suggestions have been made in this area. For an excellent review, see Truesdell and Noll (*4*, Section 96); Bowen (*5*) and Müller (*6*) consider the problems associated with a multicomponent system.

The simplest approach is to postulate that the energy flux vector $\mathbf{q}$ is a function only of the temperature T and temperature gradient ∇T,

$$\mathbf{q} = \hat{\mathbf{q}}(T, \nabla T) \tag{1}$$

The principle of material frame-indifference (*4*, p. 44) requires that Equation (1) be of the form (see Appendix*)

$$\mathbf{q} = -k\,\nabla T \tag{2}$$

where

$$k = \hat{k}(T, |\nabla T|) \tag{3}$$

The coefficient k may be referred to as the apparent thermal conductivity. Fourier's law requires that k be independent of ∇T.

Since Equation (1) was postulated, we have no way of knowing *a priori* for any particular substance the nature of the dependence of k upon T and $|\nabla T|$. We must determine this functional relationship by means of an argument based upon a molecular model for the material or by experimental measurements. By using Enskog's method for obtaining successive approximations to Boltzmann's equation, Chapman and Cowling (*7*) show that Fourier's law is valid to the third approximation for dilute simple gases whose molecules possess only translatory kinetic energy. We summarize here an experimental study of Equation (3) for solids; for details see (*8*).

EXPERIMENTAL PROCEDURE AND APPARATUS

In the experiment described below, the energy flux vector $\mathbf{q}$ was measured as a function of the temperature gradient at a given temperature.

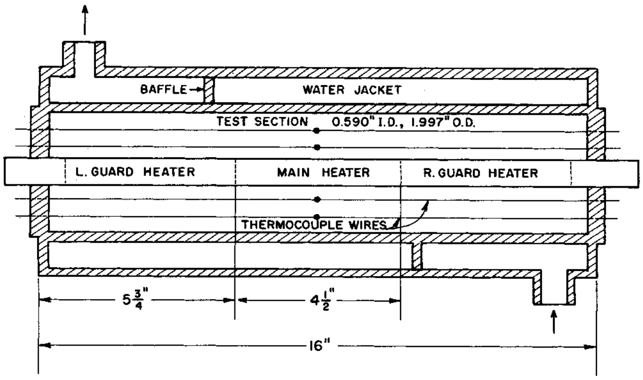

Fig. 1. Heat conduction cell and heating element.

A sketch of the apparatus is shown in Figure 1. An annular element of the solid material to be studied encloses a heating element in the center and is surrounded by a cooling jacket on the outside.

The heating element was constructed by winding nichrome wire around a ceramic cylinder. It was made in three sections: a main heater and two guard heaters. The guard heaters were used to equalize temperatures at adjacent ends of the main and guard sections in order to insure that only the radial component of the energy flux vector, q_r, was different from zero in the main section. If P represents the measured rate of energy dissipation by the main heater and L is the length of the main heater, we assume that

$$q_r = \frac{P}{2\pi r L} \tag{4}$$

The radial temperature distribution was obtained from thermocouples placed at nine different radial positions and staggered around the cell at varying values of the cylindrical coordinate θ. Since the solids studied were cured cements, the thermocouples were butt-welded and placed in the cell prior to the pouring of the uncured material. Accurate positions of the thermocouples were obtained by cutting the cells after the experiments were completed. These measurements were used to determine dT/dr at each point and consequently at each temperature.

Materials Studied

The materials used were Sauereisen cements no. 31 and 33. These are low-conductivity, amorphous materials, which can withstand temperatures of over 2,000°F. The first is classified as a high-temperature, acid-resistant cement, and the second as a high-temperature, sealing cement.

Because of their low conductivities and because of

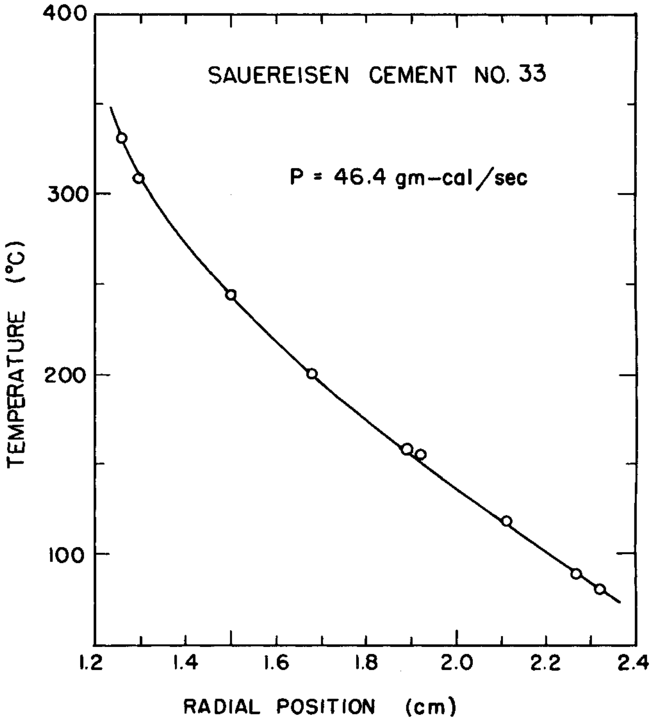

Fig. 2. A typical experimental temperature distribution.

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their abilities to resist degeneration at elevated temperatures, large temperature gradients at relatively low power requirements could be obtained in the test section.

RESULTS AND DISCUSSION

Figure 2 shows a typical temperature distribution and demonstrates the relative smoothness of the temperature-position data. It is from curves such as this that the temperature gradients were taken and used to determine k.

Figure 3 gives the dependence of k upon $(|\nabla T|)^2 = (dT/dr)^2$ for Sauereisen Cement no. 31 at three temperatures: 150°, 200°, 300°C. In the first two cases, a least-squares straight line fitted to the data shows a small negative slope; in the third case, the slope is essentially zero. Over the ranges studied, the maximum per cent change in k is 2% at 200°C.

In Figure 4, results for Sauereisen Cement no. 33 are shown for T = 100°, 200°, and 300°C. The ranges of the temperature gradient are similar to those of the previous figure. However, the results are somewhat different. For T = 100°C., we see a positive slope and a 6% variation in k over the range $0 < |dT/dr| < 150$°C./cm. The other cases indicate negative slopes and much smaller changes in k.

We question whether the changes in k with temperature gradient are due to experimental error or whether they represent deviations from Fourier's law. A conservative error analysis was made of the experiment (8, appendix B), and the error in k was estimated to be less than 5% and more probably 2 to 3%. All changes in k observed are within the experimental error with the possible exception of the data for the no. 33 cement at 100°C.

This latter case involved lower heat fluxes. In carrying out a low heat flux experiment, it was more difficult to maintain a zero temperature difference between the test section and the two guard sections. As indicated in Figure 2, the lower temperature results were taken at positions further removed from the heater, which increased the likelihood of end effects and longitudinal conduction. Consequently, we believe that systematic errors could have arisen in the measurement of the energy

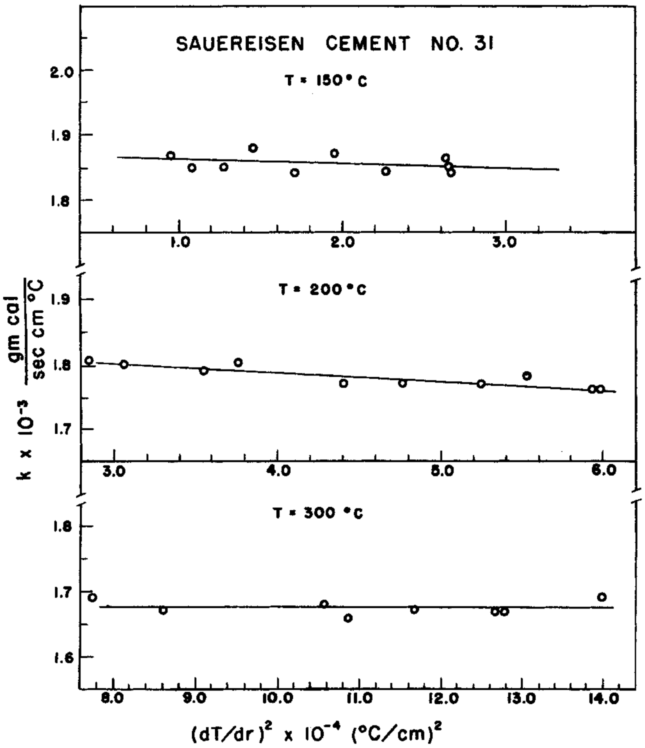

Fig. 3. The dependence of k upon $(|\nabla T|)^2 = (dT/dr)^2$ for Sauereisen Cement no. 31.

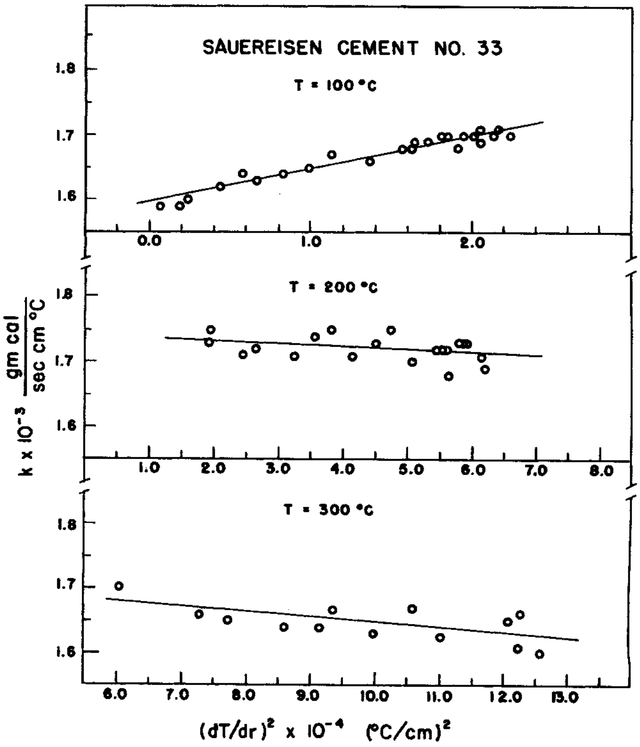

Fig. 4. The dependence of k upon $(|\nabla T|)^2 = (dT/dr)^2$ for Sauereisen Cement no. 33.

flux at 100°C. for the no. 33 cement.

Although this study is not conclusive, it may be helpful in putting the speculation about non-Fourier heat conduction in perspective. These experiments indicate that, for the two ceramic cements tested and for $|\nabla T| < 10^2$°C./cm., thermal conductivity is nearly independent of ∇T and Fourier's law represents an excellent approximation. However, one should expect effects such as these to be dependent upon molecular structure; it is possible that, when other materials are studied, more important deviations from Fourier's law will be observed.

ACKNOWLEDGMENT

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NOTATION

k = apparent thermal conductivity defined by Equation (2)
L = length of main heater
P = rate of energy dissipation by the main heater
$\mathbf{q}$ = energy flux vector
q_r = radial component of $\mathbf{q}$
r = radial cylindrical coordinate
T = temperature

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